ELASTIC EFFECTS IN THE FLOW OF POLYMER SOLUTIONS IN CHANNELS OF VARIABLE CROSS SECTION AND A POROUS MEDIUM

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The manifestation of elastic effects during the filtration of polymer solutions is analyzed theoretically.

Motion in a channel of longitudinally varying cross section is an example of how Lagrangian transience, in combination with viscoelasticity, may lead to unexpectedly strong effects. For example, in the flow of a liquid through a porous medium and in channels having a series of contractions and expansions, an anomalous increase in the pressure drop with a slight increase in flow rate is in a certain range of velocities. The pressure-drop increase may be one or two orders of magnitude and is seen even in the case of dilute solutions of polymers. Instability and hysteresis phenomena are also seen [1-3].

Below we examine these phenomena using the assumption that the intensive dilation to which elements of the liquid are subjected leads to substantial deformation of the macromolecules (oriented dilation), which entails the creation of significant elastic stresses (see [4-7]).

We will write the equations of motion in a channel in a hydraulic approximation, assuming that all of the quantities are constant over the cross section of the flow. Here, considering the effects of oriented deformation to be predominant, we ignore friction against the wall and body forces. Then we obtain [8, 9].

$$\begin{gathered} fv = q = \mathrm{const}, \\ \rho\frac{\partial vf}{\partial t} + \frac{\partial(\rho v^2 f - \sigma f)}{\partial x} = p\frac{\partial f}{\partial x}, \\ \sigma = \tau - p. \end{gathered} \tag{1}$$

We assume that the rheological behavior of the liquid is described by the model of a generalized Maxwell liquid in which τ is a single-valued function of the reversible (elastic) component of the axial dilation λ. We write the determining relations in the form

$$\frac{d\lambda}{dt} \equiv \frac{\partial\lambda}{\partial t} + v\frac{\partial\lambda}{\partial x} = \lambda\frac{\partial v}{\partial x} - \frac{\lambda}{\theta}g(\lambda), \tag{2}$$

$$\tau = G\lambda^2 g(\lambda),\ g(1) = 0,\ g'(\lambda) \geqslant 0. \tag{3}$$

Variants of rheological models of viscoelastic liquids differing in the specific form of the function $\tau(\lambda)$ and, possibly, the dependence $\theta(\lambda)$ of the relaxation time on elastic strain can be reduced to this form. The chosen form, with θ = const and g = 1, corresponds to the model of a "solution of elastic dumbbells." The study [9], where similar equations were used to analyze jet discharge and fiber formation, employed a different form more suitable for concentrated solutions and melts.

Let the area of the channel cross section change periodically along the length with the period ℓ. It is natural to assume that during steady motion (in the reference frame of the observer) in a sufficiently long channel a periodic distribution τ is established, a certain distance past the inlet section. In this case, integration of system (1) leads to the relation

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$$\frac{\Delta p}{l} \equiv -\frac{p(x+l)-p(x)}{l} = \frac{1}{l}\int\limits_{x}^{x+l}\frac{\tau}{v}\frac{dv}{dx}\,dx. \tag{4}$$

For a purely elastic liquid (more precisely, for an elastic body, $\theta = \infty$), the quantities τ and v are connected by a single-valued relation because τ is dependent only on the degree of dilation. The latter, just as velocity, is unambiguously expressed through the change in the cross-sectional area of the channel (see Eqs. (2) and (3)). In this case, it follows from (4) that $\Delta p = 0$. This conclusion, which seeems surprising at first glance, is a natural consequence of the absence of loss in the motion of an elastic body without friction through a channel of variable cross section (see [10]).

For steady motion, we find from (2) that

$$(v/\lambda)\,d\lambda/dx = dv/dx - g(\lambda)/\theta. \tag{5}$$

Multiplying this equation by $\tau(\lambda)/v$ and integrating over the period, we obtain the following from (4)

$$\Delta p = \int\limits_{x}^{x+l}\frac{\tau(\lambda)}{\lambda}\frac{d\lambda}{dx}\,dx + \int\limits_{x}^{x+l}\frac{\tau g(\lambda)}{\theta v}\,dx.$$

Due to the periodicity of the function $\lambda(x)$, the first integral is equal to zero. Then for the pressure drop we finally have

$$\frac{\Delta p}{l} = \frac{1}{l}\int\limits_{x}^{x+l}\frac{\tau g(\lambda)}{\theta v}\,dx. \tag{6}$$

Thus, to find the pressure drop it is necessary to know the distribution of λ along the channel. This can be obtained for a specified velocity field $v = v(x)$ by integrating Eqs. (2) and (3). We do this below for certain special cases. For the present we note that the integral (6) can also be evaluated without specifying the explicit form of the function $v = v(x)$. For this, we rewrite Eq. (5) in the form

$$\frac{d}{dx}\ln\frac{\lambda}{v} = -\frac{g(\lambda)}{\theta v}. \tag{7}$$

Let the velocity reach minimum values at the points $x = x_* + m\ell$ ($m = 0, \pm 1, \ldots$). Integration of Eq. (7) from x_* to x gives

$$\frac{\lambda}{v} = \frac{\lambda_*}{v_*}\exp\left(-\int\limits_{x_*}^{x}\frac{g(\lambda)}{\theta v}\,dx\right),\ \lambda_* = \lambda(x_*),\ v_* = v(x_*). \tag{8}$$

It follows in particular from Eq. (8) that by virtue of the periodicity of the flow

$$\int\limits_{x_*}^{x_*+l}\frac{g(\lambda)}{\theta v}\,dx = 0.$$

This means that in the region of periodic flow there must necessarily be sections where $\tau < 0$, $\lambda < 1$ for any flow velocity, i.e., the macromolecules are not only stretched along the flow but are even compressed. This is connected with the fact that in the dilation of the liquid in the contracting part of the channel, relaxation causes the macromolecules to be dilated somewhat less than the surrounding solvent. For the macromolecules to return to their "initial position in the solvent" during compression in the expanding part of the channel, they must undergo "reverse motion" relative to the solvent. Such motion is possible only in the compressed state.

It is natural to assume, however, that the compression relative to the equilibrium state is not great (this is shown below in several examples). Then it can be supposed that at point x_* the orientation of the macromolecules is close to equilibrium, i.e., $\lambda_* \simeq 1$, and the value of $g(\lambda)$ is no greater than unity.

In this case, with a sufficiently intensive flow, when $\theta v/\ell \to \infty$, the integral in Eq. (8) approaches zero. Thus,

$$\lambda/v \simeq 1/v_*. \tag{9}$$

Insertion of this equality into (6) gives

$$\frac{\Delta p}{l} \simeq \frac{G}{l v_*}\int\limits_{x}^{x+l}\frac{\lambda}{\theta}\,g^2(\lambda)\,dx,\ \lambda = v/v_*, \tag{10}$$

To evaluate the integral (10), we specify the value of θ by two methods: in the form $\theta = \theta_0 = \text{const}$, and in the form proposed by Hinch [5] and De Gennes [6]: $\theta = \theta_0\lambda$. The

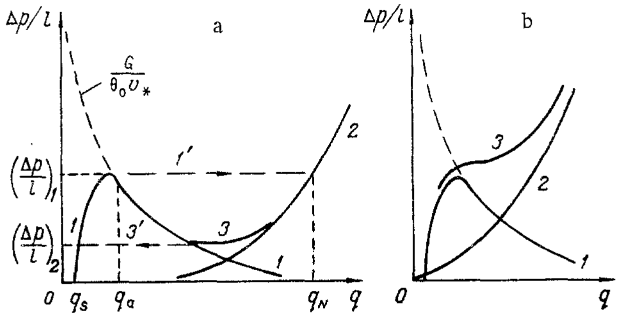

Fig. 1. Qualitative dependence of the change in pressure drop on flow rate.

specific form of the dependence of θ_0 on the molecular parameters can be found, for example, in [11].

With intensive flow, the macromolecules are in the dilated (but not to the limit) state in most of an elemental cell of the channel, which corresponds to $g(\lambda) = 1$. Then we have the following from (10) for the Hench–De Gennes case

$$\frac{\Delta p}{l} \simeq \frac{G}{\theta_0 v_*}, \tag{11}$$

while when θ is specified in the form $\theta = \theta_0$ it follows from (10) that

$$\frac{\Delta p}{l} \simeq \frac{G}{\theta_0 v_*}\left(\frac{1}{l}\int\limits_x^{x+l}(v/v_*)\,dx\right), \tag{12}$$

which is of the same order of magnitude as in (11) for flow in a channel with a relatively slight change in velocity.

It follows from Eqs. (11)-(12) that during intensive flow at a fairly high velocity ($\Theta = \theta v/\ell \gg 1$), the pressure drop decreases with an increase in the velocity of the liquid. This result expresses the transition from viscoelastic behavior of the material to elastic with an increase in flow intensity. As was shown above, losses are absent in the case of elastic behavior.

The process of transition to a regime corresponding to the above asymptotic solution can be described qualitatively as follows. The velocity of the liquid in the channel changes along the longitudinal coordinate in accordance with the profile of the channel

$$v = \frac{q}{f},\ \frac{dv}{dx} = -\frac{q}{f^2}\frac{df}{dx} = v_* f_* \frac{d}{dx} f^{-1}. \tag{13}$$

It follows from Eq. (13) that with an unaltered streamline distribution the velocity gradient increases with an increase in flow rate q (or, equivalently, with an increase in velocity v_*). This means that the flow is not intensive for fairly low flow rates, i.e., the dilation of the liquid in the contracting part of the channel occurs too slowly to take the macromolecules out of the quasiequilibrium state; here, as noted above, $\tau \simeq 0$, so that $\Delta p \simeq 0$.

The transition to intensive flow ($dv/dx > \alpha/\theta_0$) occurs when flow rate increases above a critical value $q_s = \alpha/(\theta_0(df^{-1}/dx)_{\max})$. With an increase in flow rate from q_s to a certain value q_a the flow enters a regime corresponding to the above asymptotic solutions (11) and (12), i.e., in the flow-rate interval (q_s, q_a) there is a change in $(\Delta p/\ell)/(G/(\theta_0 v_*))$ from zero to a value on the order of unity. It should be expected that in the case of a Hench–De Gennes liquid the interval (q_s, q_a) will be fairly narrow, since during intensive flow there is a sharp increase in relaxation time $\theta = \theta_0\lambda$. The latter helps the integral in (8) approach zero more rapidly with an increase in velocity.

Figure 1 qualitatively depicts the dependence of pressure drop on flow rate caused by the above-examined viscoelastic mechanism (curves 1). One feature of this dependence of the presence of a maximum and a descending characteristic. At large flow rates, when the contribution of the mechanism is small, we should expect pressure drop to be increasingly dependent on flow rate – similar to the case of a Newtonian fluid (see [3], for example).

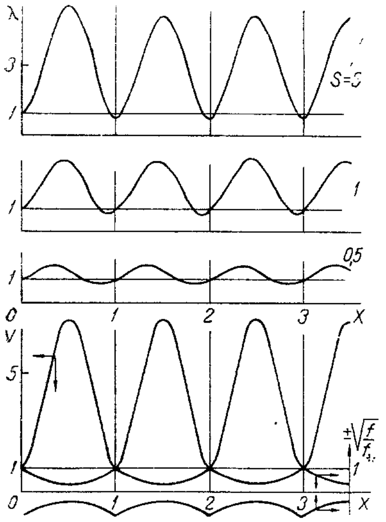

Fig. 2

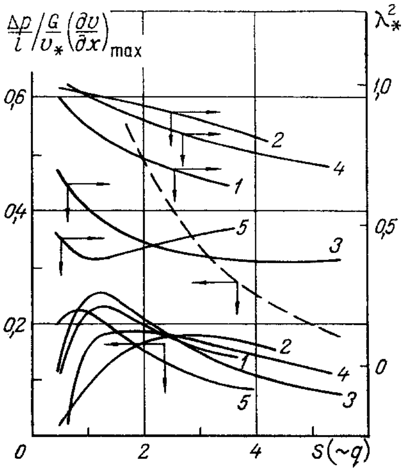

Fig. 3

Fig. 2. Results of numerical calculation of transition of flow to a regime of periodic change in the elastic strain λ along the channel for a Hench–De Gennes liquid and for different values of the intensity parameter S. Also shown is the change in the velocity and cross section along the channel.

Fig. 3. Comparison of results of asymptotic study of the problem (dashed line) and results of a numerical calculation (solid lines) of the periodic flow regime for several specific relations $V = V(X)$: 1) $V = 1 + 100 X^2 (1-X)^2$; 2) $V = 1 + 1000 X^2 (1-X)^2$; 3) $V = 1 + 10 \sin \pi X$; 4) $V = 1 + 10 (1 - \cos 2\pi X)$; 5) $V = 1 + 10 X^2 (1 - X^2)$; $0 \leq X \leq 1$.

In Fig. 1a this dependence corresponds to curve 2. The total flow-rate characteristic is shown by curve 3. Now let us assume that a flow in a channel with the flow-rate characteristic in Fig. 1a is created by a source with a specified pressure drop. Then an increase in the head from zero will be accompanied by a continuous increase in flow rate until the pressure drop reaches $(\Delta p/\ell)_1$. After this, there will be a sudden increase in flow rate, in accordance with curve 1', to the value q_N. If the pressure drop then begins to decrease, the change in flow rate will follow curve 2. Here, we may realize situations which would be impossible to obtain with a continuous increase in flow rate from zero. A further decrease in pressure leads the solution to jump to the ascending branch of curve 1. Thus, the dependence of the flow-rate change on the pressure-drop change has a hysteresislike character, which agrees with the experimental data [3]. In several cases, however, the effects of oriented elasticity may prove comparable to the effects of shear viscosity in a broad range of flow rates (which is determined by the geometry of the channel and the ratio of the viscous and elastic properties of the liquid), and the resulting pressure drop here may be a monotonically increasing function of flow rate (see curve 3 in Fig. 1b). Such a situation will obviously not lead to any hysteresis phenomena. This was evidently the situation seen in the experiments in [2].

Now let us examine the question of the effect of λ_* and present results of quantitative calculations for specific flow models.

We used the Hench model in the calculations. For this model, from (2) and (3) we have

$$\frac{d\lambda}{dX}=\frac{\lambda}{V}\frac{dV}{dX}-\frac{1}{V\Theta}\,\frac{1-\lambda^{-2}}{1-\lambda/\sqrt{N}}, \qquad (14)$$

$$X=x/l,\quad V=v/v_*,\quad \Theta=\theta_0 v_*/l,\quad g(\lambda)=(1-\lambda^{-2})/(1-\lambda/\sqrt{N}).$$

This equation was integrated numerically with the initial condition $\lambda(0) = 1$ to the point of initiation of the periodic regime for different channel profiles characterized by the relation $V(X)$ and different θ; the molecular parameter N is the number of Kuhn segments in a macromolecule and we took a value of 10^4 for it. As an example, Fig. 2 shows the dis-

TABLE 1. Comparison of Asymptotic Results with Experimental Data

Lit. source, polymer	Conc. and Reynolds number	Mol. wt. used in calculations	$S_E = \theta_0 \left(\frac{\partial v}{\partial x}\right)^{**}_{\max}$		$I_E = \frac{(\Delta p)_{\max}\theta_0 v_*}{lG}$	
			$\nu=0{,}5$	$\nu=0{,}6$	$\nu=0{,}5$	$\nu=0{,}6$
[2], polyethylene *FRA*	$c = 6\cdot10^{-6}$	$9{,}7\cdot10^6$	1,2	3,6	1173	358
	$\mathrm{Re}\in(0{,}1;\ 0{,}5)$	$1{,}0\cdot10^6$	0,04	0,1	4	1,5
	$c = 24\cdot10^{-6}$	$9{,}7\cdot10^6$	0.65	2.1	746	222
	$\mathrm{Re}\in(0{,}05;\ 0{,}2)$	$1{,}0\cdot10^6$	0,02	0,05	2,5	1,0
	$c = 96\cdot10^{-6}$	$9{,}7\cdot10^6$	0.4	1,3	613	191
	$\mathrm{Re}\in(0{,}01;\ 0{,}1)$	$1{,}0\cdot10^6$	0,01	0,03	2,1	0,82
[2], polyethylene *WSR* = 301	$c = 20\cdot10^{-6}$ $\mathrm{Re}\in(0{,}2;\ 0{,}3)$	$4{,}2\cdot10^6$	0,77	2.2	12	4
	$c = 80\cdot10^{-6}$ $\mathrm{Re}\in(0{,}1;\ 0{,}2)$	$4{,}2\cdot10^6$	0,38	1,1	3,90	1,3
	$c=160\cdot10^{-6}$ $\mathrm{Re}\in(0{,}06;\ 0{,}1)$	$4{,}2\cdot10^6$	0,27	0,8	0,7	0,23
[3], polyethylene *WSR*=301	$c=50\cdot10^{-6}$ $\mathrm{Re}\in(80;\ 100)$	$4{,}2\cdot10^6$	0,37	1,1	3,54	1,2
	$c = 100\cdot10^{-6}$ $\mathrm{Re}\in(10;\ 20)$	$4{,}2\cdot10^6$	0,27	0,8	0,7	0,23
	$c = 200\cdot10^{-6}$ $\mathrm{Re}\in(500;\ 1000)$	$4{,}2\cdot10^6$	0,69	2,03	25,6	8,7

tribution of $\lambda(X)$ for different flow intensities S in a channel with the distribution $V(X) = 1 + 100\, X^2\, (1 - X)^2$. As in the other results presented below, $S = \Theta\max V'(X)$. The calculations also confirm the closeness of λ_* to unity in expansions of pore channels. A rapid transition to the periodic regime is typical. The pressure drop for the periodic solution was found from Eq. (4).

The dependence of the ratio $\Delta p/\ell$ on flow intensity S is shown in Fig. 3 for different channel profiles. While differing in their details, these dependences have a common feature: The presence of the maximum at $S \simeq 1$. Asymptotic formula (11), the results of calculations with which are shown by the dashed line in Fig. 3, qualitatively correctly describe the behavior of the solution at $S > 2$.

Also shown are values of strain λ_* reached at the point of the minimum velocity x_*. The fact that the actual value of λ_* is somewhat less than unity leads to exaggeration of the results calculated from asymptotic formula (11).

Let us compare the results obtained with available empirical data from [2, 3]. The comparison will be performed according to the values

$$S_E = \theta_0\, (\partial v / \partial x)^{**}_{\max},\ I_E = ((\Delta p)_{\max} / l)/(G / (\theta_0 v_*)). \tag{15}$$

Here, $(\partial v/\partial x)^{**}_{\max}$ is the value of the maximum velocity gradient along the path at which the relationship between the pressure drop and flow rate deviates from linearity; $(\Delta p)_{\max}$ is the pressure-drop maximum. Here, for tests with a layer of spherical particles we took $v_* \simeq 2.2u$, $(\partial v/\partial x)_{\max} = 43.5u/d$, where d is the diameter of the spheres. For the channel with periodic contractions $v_* = q/f_*$, $(\partial v/\partial x)_{\max} = 8q/(\pi d^3)$, d is the diameter of the contraction.

The rheological parameters θ_0 and G were evaluated from the formulas [5, 6]:

$$\theta_0 = 6\pi\mu r_0/\varkappa,\ \varkappa = 3kT/(Nb^2),\ G = n\varkappa r_0^2. \tag{16}$$

Here, to determine the unperturbed radius of a macromolecule in a solution r_0 we used the relation [11]

$$r_0 = bN^{\nu} \tag{17}$$

with $\nu = 0.5$ (ideal chain) and $\nu = 0.6$ (real chain).

The theoretical values of S_E and I_E have the order 1; the experimental values calculated with Eqs. (15)-(17) are shown in Table 1.

The main difficulty in making a comparison is in the indeterminateness of the molecular mass M, which heavily affects the value of I_E (as $M^{5/2}$). It can be concluded from an analysis of the tabular data that the initiation of the effect of an anomalous increase in pressure is connected with the longest molecules, while the maximum of the effect is due to the shorter molecules which comprise the bulk of the polymer. It follows from Figs. 2 and 3 that in the region of the maximum of the phenomenon of anomalous pressure increase the relative elongations of the macromolecules are not so great (reaching several units), while the additional increase in pressure is of the order of G in terms of the period of the medium ℓ. For flow in a porous medium of permeability k, we have the estimate $\ell \sim \sqrt{k}$. The additional pressure gradient due to elasticity is approximately $G/\sqrt{k}$, while the main gradient ("viscous") is about $\mu u/k$. According to the estimates made, at the maximum of the effect $u \sim d(\partial v/\partial x)_{\max}$, $(\partial v/\partial x)_{\max} \sim 1/\theta_0$. From this

$$u \sim d/\theta_0,\ \mu u k \sim \mu d/(k\theta_0).$$

Thus, the ratio of the additional pressure drop to the main pressure drop is of the order

$$Gk\theta_0/\sqrt{k}\,\mu d \sim nr_0^3.$$

Thus, the additional increase in pressure due to elastic effects is asymptotically independent of the permeability and is proportional in a dilute solution approximation to the fullness of the volume with macromolecular clusters; the resistance maximum is reached at filtration velocities $\sqrt{k}/\theta_0$ which are lower, the lower the permeability of the medium.

NOTATION

t, time; x, longitudinal coordinate of channel; v, velocity of liquid flow; f, cross-sectional area of channel; x_*, longitudinal coordinate corresponding to maximum cross section of channel; v_*, f_*, velocity and cross-sectional area at this point; ρ, density of liquid; q, volumetric flow rate; p, pressure; σ, axial component of stress tensor; τ, axial component of tensor of excess stresses; λ, elastic elongation; θ, relaxation time; G, elastic modulus; g(λ), function accounting for the absence of stresses in the liquid in the equilibrium state and nonlinear effects of the finite extensibility of the material; Δp, pressure drop on one element of the channel; ℓ, length of an elemental cell of the channel; $m = 0, \pm 1, \ldots$, whole number; α, number on the order of unity; $X = x/\ell$; $V = v/v_*$; $\Theta = \theta_0 v_*/\ell$; μ, shear viscosity of liquid; b, length of Kuhn segment; N, number of Kuhn segments in a macromolecule; kT, Boltzmann temperature; κ, elastic constant of a macromolecule; n, number of macromolecules per unit volume; r, distance between ends of macromolecule; r_0, size of macromolecule in equilibrium conformation; d, diameter of spheres used to model a porous medium or diameter of a contraction in an axisymmetric channel; u, specific flow rate; $S = (\partial v/\partial x)_{\max}\theta_0$, index of flow intensity; c, polymer concentration; M, molecular weight of polymer; k, porosity of medium.

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RECONSTRUCTION OF BED PRESSURE WITH DATA OF NORMAL WELL OPERATION

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The article suggests an algorithm for reconstructing the bed pressure from measurements of pressure and yields of wells in the course of their normal operation.

In solving the problem of developing petroleum deposits it becomes necessary to check changes of pressure in the oil bearing bed. The existing methods consist in estimating the bed pressure, i.e., the pressure that would establish itself at the bottom of a shutdown well after complete stoppage of inflow of liquid. At present this value is determined chiefly by a depth manometer in the course of unsteady investigations involving shutdown of the well. In practice, it is often impossible or undesirable to shut down an operating well. Methods are therefore worked out which make it possible to estimate the bed pressure from data of normal well operation.

All the known methods of estimating bed pressure are in essence methods of identificationary nature, and they are not suitable for reconstructing the pressure distribution near an operating well. The lack of this kind of information complicates the mathematical modeling of the processes of oil extraction in the sense that it prevents the solution of nonsteady filtration in the traditional initial and boundary statement. Without an operative estimate of the pressure distribution in the bed it is difficult to state the initial conditions because for that it is necessary to solve successively a considerable number of normal problems where the result of the preceding problem is taken as the initial condition for each problem upon change of the operating regime.

In the present work the problem is reduced to restoring the initial condition (and thus also the subsequent states) for an equation of parabolic type from redefined boundary conditions. It is assumed that the time of piezoconductivity and the hydroconductivity of the bed are specified (they may be, e.g., determined previously in the course of nonsteady investigations of wells [1]). The stated problem belongs to the class of inverse retrospective problems, and as is well known, it is a malposed problem. It is solved by the method of ordered minimization of the mean risk [2]; this makes it possible under conditions of the static approach to obtain guaranteed solutions (with a certain probability) from a limited amount of empirical data.

Nonsteady filtration of petroleum in a well (s = 1) or in a gallery of wells (s = 0) is described by the equation

$$\frac{\partial p}{\partial t} = Lp \equiv \frac{1}{x^s} \frac{\partial}{\partial x} \left(x^s \frac{\partial p}{\partial x} \right), \; x_0 < x < 1, \; 0 < t < \infty, \tag{1}$$

$$p(x, 0) = \varphi(x), \tag{2}$$

$$p(x_0, t) = \psi(t), \tag{3}$$

$$\frac{\partial p(1, t)}{\partial x} = 0, \tag{4}$$

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